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Early Neutron-Star Matter ^{*)}
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In the interior of cold (and old) neutron stars, matter is in its lowest energetically possible groundstate (apart from possible frozen sheer-tensions, assuming a solid neutron core) with densities not more than, say, a factor of ten deviating from the nuclear saturation density in nuclei of $2 \cdot 10^{14} \text{ gr/cm}^3$.

Years ago [1] are the times of rough estimates of some gross properties (equation of state, composition), first by macroscopic semi-empiric models, later on by phenomenological microscopic models like Thomas-Fermi or other nuclear-theory models assuming simple semi-empiric fitted two-body forces.

At present, for more sophisticated questions (solidification to a neutron quantum-crystal, nonexistence of a pion-condensate, nonequilibrium transport-properties like cooling, speed-up healing) more sophisticated nuclear physics is needed, which may be less arbitrarily extrapolated from studying the known nuclei of finite atomic number A and small relative neutron-excess $I := (N-Z)/A \lesssim 0.24$ or even scattering of free nucleons, to neutron-star matter of $I \approx 0.8$, or its extremely neutron-rich nuclei in the inner part of the outer crust.

What is necessary, is the best possible taking care of

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the contributions of the p-n-n interaction, of the probably most important $\Delta := n_{33}$ -resonance contribution, as was elucidated in the preceding talk of G.E. Brown. Not before a realistic meson-theoretically founded nucleon-nucleon force and a safe method to treat the many-body problem are applied, will reliable results be obtained.

Early neutron-star matter, that is matter of the interior core of a supernova, collapsing to a neutron star gets increasingly dense (from 10 to 10^{15} gr/cm³), - thus calling for a treatment of the nucleon-nucleon interaction over a wide density range, - and is very hot (from 10^9 to 10^{11} K), which is comparable even with the nuclear Fermi-energy.

Thus for these astrophysical applications one has to tackle the problem of how to treat hot nucleon-matter and hot nuclei. This will be a new field for future topics of sophisticated nuclear theories. Especially, because of the temperature-caused distribution up to high nucleon velocities, a precise knowledge of the dependence of the nucleon-nucleon force on high relative momenta is necessary. Therefore here the effects of the n_{33} -resonance and of the mesonic properties of the two-body force will be much more pronounced than for zero-temperature nucleonic matter or nuclei.

In this contribution dirty guesses only, and calculations on a simple basis are presented, together with a survey of those astrophysical processes in a supernova, where the properties of hot nucleonic matter or nuclei may enter most

dramatically, which are

1. the formation of the neutron-rich nuclei of the inner neutron-star crust,
 2. the disintegration of nuclei during the compression phase of the supernova,
 3. the thermostatic properties of hot neutron-star matter,
- where the order of these three topics is chosen here, reflecting the increasing complexity of the nuclear model applied here:

- 1) freezing of nuclear transfer-reactions,
- 2) temperature-dependent binding energy of known nuclei, and
- 3) thermostatics of hot nucleonic matter of any proton to neutron ratio, and extremely neutron-rich nuclei, within the frame of the Thomas-Fermi model.

Q. ground-state nucleon-matter and nuclei

The properties of matter in its energy-groundstate at high densities is now shortly discussed to serve as a basis for the later chapters.

At temperatures far below the chemical potential ($T < \mu$), which is proportional to the density as $\rho^{1/3}$, the thermostatic properties such as the composition, binding, pressure, do not depend on temperature but on density only, as is known e.g. for strongly degenerated Fermi-systems.

If the matter is assumed to be completely catalyzed (no geological, chemical, nuclear reactions hindered), then its properties should be those of matter at the same density but

in its energy-groundstate, which has been evaluated to be as follows: for densities up to $\rho_1 = 10^6 \text{ gr/cm}^3$, a ^{56}Fe -crystal; then up to $\rho_2 = 10^{11} \text{ gr/cm}^3$ more and more relatively neutron-rich nuclei with $Z=28$, then $N=50, 82$. At ρ_2 the neutrons start to leak out of the nuclei (the chemical potential of the neutrons getting positive), thus extremely neutron-rich nuclei of $Z \lesssim 38$ are formed, which are embedded in an increasingly dense free neutron gas; finally above $\rho_3 = 10^{14} \text{ gr/cm}^3$ a homogenous neutron-matter with an admixture of about 10% protons will form the bulk of the inner part of the neutron star. For heavy stars there may exist either a neutron quantum-crystal or an hadronic core.

The quantitative predictions depend on the nuclear method applied. The progress involved may be read off from fig.1, which shows the river of knowledge broadening towards the future, like the river Rhine towards its estuary. The time-direction should indicate also the trend towards more powerful, more elaborate, and more fundamental methods. Noted are those nuclear properties, which are described by the respective methods.

The liquid drop model of v.Weizsäcker [4] and Bethe [5] describes the nucleus as being a charged incompressible drop of two completely mixing "liquids" with a total density ρ_c independent of charge Z and mass number A of the nucleus. The four free parameters (ρ_c , volume-, surface-tension, asymmetry-coefficient) may be adjusted, as good as possible in this simple semi-empiric mass formula, to the experimentally known

nuclear masses, yielding a mean deviation of about 4 MeV ($\approx 1\%$).

In contrast to the semi-empiric LDM, the Droplet Model of ref. [6] has been constructed to be the analytic extrapolation for large A, small deformations, and small deviations from symmetry and saturation density,

$$(1) \quad \bar{\delta} := \frac{\bar{S}_n - \bar{S}_p}{\bar{S}_n + \bar{S}_p} \sim I := \frac{N-Z}{A} \ll 1, \quad \bar{\epsilon} := \frac{S_s - S_0}{S_0} \ll 1, \quad A^{-1/3} \ll 1, \quad \frac{t}{R_c} \ll 1,$$

(with t being the surface thickness, R_c the radius of the nucleus), of the simple microscopic and selfconsistent Thomas-Fermi model for two completely mixing Fermisystems, assumed to be leptodermous. The DM contains, in addition to the LDM, terms, which describe

1. the compressibility of the nucleus, by which radii and central densities depend on the balance of the Coulomb-repulsion and surface-tension compression,
2. surface-symmetry and volume-symmetry terms, dependent on $\bar{\delta}$.

The DM is evaluated by starting from an energy-density functional $e(\delta, \epsilon)$, which is evaluated to second order at $\delta = \epsilon = 0$.

Adjusting either the two-body force parameters in the Thomas-Fermi model [8], or some adjustable parameters in the resultant DM-mass formula itself [10], yields

1. a mean (rms) - deviation of experimental nuclear masses to theoretical predictions to 1.0MeV,
2. the neutron- and proton radii of known nuclei, including the experimentally known radius isotope-shift anomaly, (adding a neutron increases the radius of a nucleus only by 40 % of the value, predicted by the LDM),
3. the prediction of a neutron skin $(R_n - R_p) > 0$. Although the experiments up to now were not of that precision as to prove or

disprove the quantitative prediction of the DM, many exp. results could be understood on the grounds of the existence of a neutron-skin [7].

The Thomas-Fermi model can also be used directly to calculate the density distributions and masses of nuclei. But since there are no constrictions of the type of equation (1), it can be applied directly for the astrophysical applications such as extremely neutron-rich nuclei ($\bar{\delta}$ not small), large deformations (fission barriers, needed in the termination of the r-Process path), nuclei embedded in a neutron gas, and even pure neutron matter.

The results may depend on the adopted two-body force. H. v. Groote used the phenomenological proposition of Seyler and Blanchard: an Yukawa-type attraction in space with a repulsive part, proportional to the square of the relative momentum. The 4 free parameters were adjusted [8], so that the experimentally found binding of known nuclei came out better than 0.4 % (see above). For his subsequent calculation of the surface tension of nuclei of given bulk asymmetry, the results [9] are plotted in fig. 2. Clearly the LDM is right only for zero asymmetry whereas the DM, which contains a surface-symmetry term proportional to $\bar{\delta}^2$, exhibits to be the correct approximation of the Thomas-Fermi model for small $\bar{\delta}^2$. For larger $\bar{\delta}^2$ it predicts however a pronounced enhancement of surface-energy compared to the DM. This peak goes with a peaked neutron-distribution skin-thickness. Beyond the neutron-drip line the nuclei are embedded in a free neutron gas, and in the peak-area the neutron-distribution is much smoother going from the nuclear value to the free gas value, the proton-density goes to zero. For $\bar{\delta}^2 \gtrsim 0.2$

the protons leak out of the nucleus as well and the selfconsistent calculation results in a homogenous nucleon-matter.

The Thomas-Fermi model seems to be well suited to calculate the gross groundstate-properties of nuclei of any asymmetry, and might be useful to calculate the thermostatic properties of heated nuclei or nucleon matter, once the forces were known. An outline of the model is given in sect.3.

1. matter in early neutron stars

One of the astrophysical objects, where the properties of hot nuclei and their nuclear transfer-reactions have to be known, is the early stage of a neutron star. For it is difficult to imagine, how during the formation of the star, with matter being compressed from 10^3 gr/cm^3 to $10^6 - 10^{11} \text{ gr/cm}^3$ and cooled down from 10^{11} K to 10^7 K , the matter could possibly transmuted to the groundstate, since multi-nucleon transfer-reactions would then be necessary, which are at low temperatures largely frozen because of the high Coulomb-barrier. Thus one would expect the neutron-star crust-matter to be in a metastable state.

For a quantitative calculation of its actual composition and equation of state one would be in need of the knowledge of

1. the hydrodynamical path of each piece of matter, which ends up in the neutron-star crust, with respect to density and temperature as a function of time,
2. the cross-sections of all transfer-reactions as a function

of T and density,

3. a numerical reaction-network calculation which follows the nuclear transmutations of each piece of matter during its hydrodynamical history. Such programs are in progress, but have not been carried through up to now.

A qualitative orientation one might gain from the following considerations: assume that the hydrodynamical time-scale is long, compared to the β -decay live-times. Then only the transmutations between β -stable nuclei have to be considered. In fig.3 the bottom of the β -valley is plotted as a function of A for a set of baryon number-densities. Clearly for the nucleus of groundstate matter the A can be read off as the minimum of the respective density-curve. The result is plotted in fig.4a.

The asymptotic equilibrium nuclear abundance-distribution for finite temperature for hydrodynamical timescales long compared to any transfer-reactions would be a distribution like $\exp(-f(A,Z)/T)$. Less bound nuclei would be less prevalent. If however the temperatures are low, and transfer of heavier particles extremely unlikely, and the time-scale is short, then the resultant distribution will depend on the historical seed-nuclei distribution. The barriers in the bottom of the respective β -valley may not be surpassed even if there exists a deeper minimum for heavier nuclei.

For some simple model-assumptions the results are given in fig.4b) and c):

1. cold compression of matter (squeeze a piece of ^{56}Fe to densities of 10^{11}gr/cm^3). Then no transfer-reactions are possible,

and the Fe will simply undergo the β -decay to stay in the β -valley corresponding to the momentary density up to the n-drip.

2. low temperature squeezing, where only transfer of neutrons is likely (no Coulomb-barrier),
3. higher temperature squeezing, where in addition transfer of α -particles and protons is possible.

The results are:

1. the charge of the resultant nuclei does not increase with density as compared to ground-state-matter,
2. the equation of state is stiffer, the n-drip occurs earlier, the sheer strength of the crust is less in all three cases compared to the respective ground-state.

2. Disintegration of nuclei

The collapse of the supernova-core to a neutron star is triggered by the cooling due to the disintegration of Fe-nuclei. Thus both the temperature at which the disintegration takes place and the latent heat used are essential input parameters of the hydrodynamical supernova model-calculations. Up to now it has been assumed, with one widely known exception [14] of a preliminary consideration, that ground-state Fe-nuclei disintegrate. But the collision of Fe-nuclei, establishing the gas-kinetic equilibrium, may well be inelastic, thus exciting the Fe-nuclei, so that this inner degree of freedom, the excitation, on the average will have the same

temperature. Thus we have to calculate the binding energy of nuclei

$$B(N, Z, T)$$

as a function of temperature. Using the experimentally known nuclear excitation-levels of a set of Fe-group nuclei and taking into account the partial degeneracy of the neutron-gas, M.El Eid [15] has written the appropriate numerical program. Qualitatively the main features of the expected results might be read off by some model-calculations.

1. Let us restrict to a system of the components $\{Fe, \alpha, n, p, e^-, e^+\}$. Then the resulting lines with ground-state Fe^{56} , are plotted in fig. 5, each line defining the temperature- and density pairs where nucleons in two components are equally abundant. For low temperatures and density we have mostly Fe, for higher temperatures mostly α , and for high densities and temperatures mostly n.
2. If in the calculation Fe^{56} is replaced, just for studying the effect, by a model-nucleus of Fe^{*56} assumed to be in an excited ($E^* = 3MeV, J = 1/2$) level, then the (Fe - α) disintegration line is at lower temperatures; being less bound, it disintegrates at lower temperatures. see fig. 5.
3. If we take the whole set of excitations of the Fe^{56} , this effect is offset by the large increase of the phase space, so that in a system $\{Fe, Fe^{*1}, Fe^{*2}, \dots, n, p, e^-, e^+\}$ the total iron number-density will decrease because of disintegration at a somewhat higher! temperature. The argument for that is the same as for the disintegration itself: For thermostatic equilibrium at a given temperature and density, not the total

energy density, but the free energy density

$$f = e - T \cdot s$$

has to be a minimum. A disintegration is increasing e , but the entropy-density is largely increasing, s being proportional to the logarithm of the allowed phase-space, which is a product of the phase-spaces of each individual components, thus s increases largely with the number of components. So, if T is sufficiently large, then f is lowered even if e is larger, for disintegration and analogously for allowing excited states of Fe to be populated.

3. Hot nucleonic systems

The average nuclear binding $B(N, Z, T)$ for a given (high) temperature, not knowing the exp. excitation spectra, for e.g. very neutron-rich nuclei, heavy-ion reaction-fragments or neutron matter may be calculated microscopically by first subtracting the shell-effects, assumed to be washed out for higher temperatures,

$$B(N, Z, T) = \bar{B}(N, Z, T) + B_{shell}(N, Z, T).$$

For $\bar{B}(N, Z, T)$ only a largely simplified calculation of Stocker [16] exists, starting from the free energy density $f = e - T \cdot s$ for a Fermi-system to be minimized, but assumed, that the entropy density $s(\rho, T)$ would be the same as for a non-interacting Fermi-system, $s \approx s_0$, and that the energy-density $e(\rho, T)$ would be the same as for a cold system, but inserting the density-temperature dependence of a noninteracting Fermi-system.

A general systematic and selfconsistent approach to calculate the thermostatic properties of heated Fermisystems of interacting nucleons was presented by W.A.Küpper, G.Wegmann et al. [13]. Here just a short review is given:

A system of nuclei, where the excitation is a degree of Freedom, in thermal equilibrium with a heat bath at temperature T will emit and absorb neutrons and protons, so that in equilibrium the nucleus will be immersed in a free gas of neutrons and protons, although their densities are rather small for low temperatures, and thus the thermal relaxation time may become very large.

Assume the nucleus to be a system of two interacting Fermi-liquids. Then the total number of nucleons and their energy in a given volume \mathcal{V} will be

$$(a) \quad A = \sum_{\tau} 2 \cdot \int_{\mathcal{V}} d\underline{r} \cdot \rho_{\tau}(\underline{r}) = \sum_{\tau} \int_{\mathcal{V}} d\underline{r} \frac{2}{(2\pi)^3} \cdot \int d\underline{k} \cdot \rho_{\tau}(\underline{k}, \underline{r}),$$

$$(b) \quad E = \sum_{\tau} 2 \cdot \int_{\mathcal{V}} d\underline{r} \cdot \varepsilon_{\tau}(\underline{r}) = \sum_{\tau} \int_{\mathcal{V}} d\underline{r} \frac{2}{(2\pi)^3} \cdot \int d\underline{k} \cdot \rho_{\tau}(\underline{k}, \underline{r}) \cdot \left[\frac{\hbar^2 k^2}{2m_{\tau}} + \frac{1}{2} V_{\tau}(\underline{k}, \underline{r}) \right],$$

where $\tau := \pm 1$ is the isospin. $\rho_{\tau}(\underline{r})$ and $\varepsilon_{\tau}(\underline{r})$, the density and energy-density of component have been replaced by an integral over the Wigner-transforms of the mixed density and potential, defined by

$$(c) \quad S(\underline{r}, \underline{r}') := \frac{1}{(2\pi)^3} \cdot \int d\underline{k} \cdot e^{i\underline{k}(\underline{r}-\underline{r}')} \cdot S\left(\frac{\underline{r}+\underline{r}'}{2}, \underline{k}\right),$$

and analogously for $V_{\tau}(\underline{k}, \underline{r})$.

An ansatz for the potential function $V_{\tau}(\underline{k}, \underline{r})$ is

$$(d) \quad V_{\tau}(\underline{k}, \underline{r}) = \frac{2}{(2\pi)^3} \cdot \int d\underline{r}' \int d\underline{k}' \cdot [V_p \cdot \rho_{\tau}(\underline{k}', \underline{r}') + V_n \cdot \rho_{-\tau}(\underline{k}', \underline{r}')],$$

where V_p ($|\underline{r}-\underline{r}'|, |\underline{k}-\underline{k}'|$) give the interaction of (un)like par-

ticles and depend on the distance and relative momentum of the two particles. For actual numerical calculations W.Küpper used the Seyler-Blanchard potential

$$(e) \quad V_e = C_e \cdot \exp(-|r-r'|/r_D) \cdot (1 - |k-k'|^2/k_D^2) / (|r-r'|/r_D),$$

being quadratically dependent on $|k-k'|$, the parameters r_D, k_D, C_e, C_e' of which had been fitted to exp. groundstate masses [8]. However, because of the high-energy "Boltzmann-tail" in the Fermi distribution the high-temperature results are rather sensitive to k_D and the ansatz (e) may not be on good grounds.

Generally the entropy can be written for a Fermi liquid system as

$$(f) \quad S := \sum_{\tau} \int d\mathbf{r} \frac{2}{(2\pi)^3} \cdot \int d\mathbf{k} \left(s_{\tau}(k, r) \cdot \ln s_{\tau}(k, r) + (1 - s_{\tau}(k, r)) \ln(1 - s_{\tau}) \right).$$

Thus in principle we have from (b) and (f) the free energy

$$F := E - TS,$$

which is a thermodynamic potential from which all desired thermodynamic properties can be derived and calculated.

The Thomas-Fermi approximation now is the choice

$$(g) \quad s(k, r) = \left\{ 1 + \exp \left[\frac{1}{T} \left(\frac{\hbar^2 k^2}{2m_{\tau}} + V_{\tau}(k, r) - \mu_{\tau} \right) \right] \right\}^{-1},$$

with μ_{τ} being the chemical potential.

Specializing for homogenous nucleon systems (d) simplifies to $V_{\tau}(k)$ for which an ansatz, inserting (e) into (d) can be made:

$$(h) \quad V_{\tau}(k) =: V_{0\tau} + k^2 \cdot V_{1\tau}.$$

Defining an effective mass, which does not depend on the temperature, by

$$(i) \quad \frac{\hbar^2}{2m_\tau^*} := \frac{\hbar^2}{2m_\tau} + V_{1\tau},$$

one gets from (g)

$$(g) \quad g_\tau(k) = [1 + \exp(x - \eta_\tau)]^{-1},$$

where the abbreviations

$$(k) \quad x := \frac{\hbar^2 k^2}{2m_\tau^*} / T, \quad \eta_\tau := (\mu_\tau - V_{0\tau}) / T$$

have been used.

The results for the case of the Seyler-Blanchard two-body potential is

$$(l) \quad g_\tau(\mu_\tau, T) = \left(\frac{2m_\tau^* T}{\hbar^2} \right)^{3/2} \cdot I_{1/2},$$

$$(m) \quad f(\mu_\tau, \mu_{-\tau}, T) = \sum_{\tau} g_\tau \left\{ T \cdot \left(\eta_\tau - \frac{2}{3} \frac{I_{3/2}}{I_{1/2}} \right) - \pi^2 k_D^2 \cdot (\alpha g_\tau + \beta g_{-\tau}) \right\}$$

with the Jüttner-integrals

$$(n) \quad I_\nu(\eta) := \int_0^\infty dx \cdot x^\nu \cdot g_\tau(x - \eta).$$

From (m) all thermostatic quantities can be calculated, if the results of (l) are inserted, to yield f , which is a thermostatic potential, $f(p_\tau, p_{-\tau}, T) = f(\mu_\tau(p_\tau, T), \mu_{-\tau}(p_{-\tau}, T), T)$.

Some results for the homogenous nuclear matter $p_\tau = p_{-\tau}$ are presented here, taken from ref. [13].

The phase transition behaviour of hot nuclear matter is plotted in fig. 6 and compared with the principle of corresponding states. Its clearly seen, that the behaviour of the Thomas-Fermi model nuclear-matter has not the same critical index (exponent of $(T - T_c)$) as the exp. systems. This deficiency is similar to e.g. v.d.Waals and other Theoretical approaches.

The effective mass does not depend on temperature. Its den-

sity dependence as given in fig. 7, seems to be too steep which may stem from the strict $|k-k'|^2$ -dependence of the two-body potential.

The specific heat per nucleon, as seen from fig. 8 shows a familiar behaviour, being linear for low temperatures and $3/2$ for the high temperature-low density limit.

Finally, the latent heat of the transition is given in fig. 9. Most qualitative behaviours of hot nuclear matter including the absolute magnitude of $T_c \approx 17$ MeV has been gained by Palmer [17], by scaling the He^3 two-ion-potential to the nucleon-nucleon interaction dimensions.

For making available prior to publication some results of numerical calculation contained in the figures, and a joyful collaboration, I wish to thank Dipl.-phys. M.F. El Eid, Dipl.-phys. H. v. Groote, Dr. K. Koebeke and Dipl.-phys. W.A. Küpper. And for valuable remarks and elucidating discussions I wish to thank Dres. K. Takahasi and W. Hillebrandt.

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figure captions
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fig. 1: nuclear properties, which have been taken care of by the named models.

fig. 2: surface tension of heavy neutron-rich nuclei as a function of the proton-neutron asymmetry $\bar{\delta} := (\rho_n - \rho_p) / (\rho_n + \rho_p)$ in the center of the nucleus. Beyond $\bar{\delta}^2 = 0.086$ the nuclei are embedded in a free neutron gas (fig. taken from H. v. Groote, ref. [9]).

fig. 3: The bottom of the β -valley (β -stable nuclei) for a given set of baryon-number densities as indicated. Plotted is the free-energy per nucleon minus restmass versus mass-number A . The line for each density has its own ordinate (figure taken from K. Koebke, ref. [12]).

fig. 4: Charge Z and number of neutrons N of β -stable nuclei in matter of density $10^6 \text{ gr/cm}^3 \lesssim \rho \lesssim 10^{11} \text{ gr/cm}^3$ (increasing density indicated by an arrow).

In fig. a) matter is assumed to be in its completely catalyzed groundstate; in fig. b) those nuclei, which can be reached by cold compression of ^{56}Fe , ^{82}Ge and ^{122}Zr , are plotted, in fig. c) the compression is assumed to take place at low temperatures, where either no charged particles (+), or only particles with $A \lesssim 4$ (●) can be transferred by nuclear transfer-reactions, others are being frozen due to the high Coulomb-barrier. (Figure taken from K. Koebke, ref. [12]).

fig. 5: The densities and temperatures are plotted, where in a mixture of $\{ ^{56}\text{Fe}, ^4\text{He}, ^1\text{H}, n \}$ the ratio of two abundances is proportional to the number of nucleons in the respective nuclei: full dots represent $[56 \cdot ^{56}\text{Fe} : n = 1]$, crosses $[4 \text{ He} : (n \text{ H}) = 1]$, and circles $[56 \cdot ^{56}\text{Fe} : 4 \cdot \text{He} = 1]$, with the iron nucleus assumed to be excited to the kinetic temperature of the nuclei. Clearly, in the high density region, neutrons are prevalent, whereas, in the high-temperature region, ^4He prevails up to $1,4 \cdot 10^{10} \text{ K}$. A less bound nucleus disintegrates at lower T compared to a given nucleus (here $^{56}\text{Fe}^* : ^{56}\text{Fe}$) (numerical calculations done by M. El Eid).

fig. 6: Densities and temperatures of the liquid-gaseous phase transition in units of the critical point values. The lines for all classical inert gases (Ne, Ar, Kr, Xe, N₂, O₂, CH₄) coincide, in accordance with the principle of corresponding states. The experimental results for the quantum-gases He³, He⁴ are given. The theoretical results of Thomas-Fermi model for nuclear matter and of v.d.Waals for classical gases is plotted. Their critical exponent is different of the respective experimental results (figure taken from W.A. Küpper, ref. [13]).

fig. 7: The effective mass m^* -to mass m -ratio is plotted versus density. It does not depend on temperature (fig. taken from W.A. Küpper, ref. [13]).

fig. 8: Specific heat per nucleon $c_v/n := T \cdot \partial_T^2 \epsilon$ of nuclear matter plotted versus temperature for a set of densities as indicated, n_0 being the ground-state saturation-energy. For low temperatures compared to the chemical potential the specific heat is linear, for high temperatures it finally tends to the wellknown 3/2. (figure taken from W.A. Küpper, ref. [13]).

fig. 9: The latent heat of vaporization $\mathcal{L} := T \cdot (\lambda_v - 1)$ plotted versus temperature. (figure taken from W.A. Küpper, ref. [13]).

River Rhine

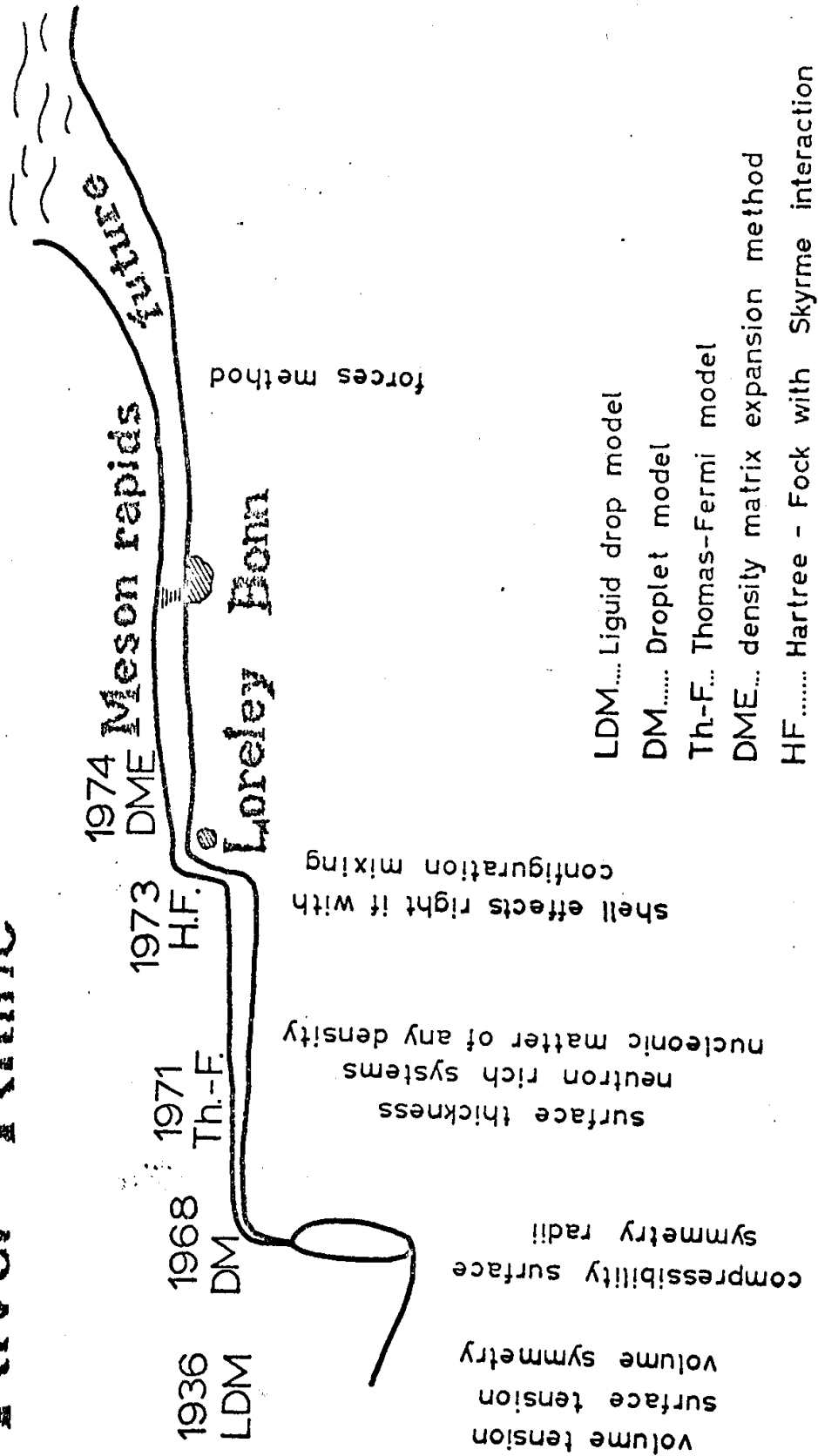


fig. 1

