Shell structure and stability
of very neutron-rich isotopes

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This is to point out the aim and to explain the interest in the question of how the single-particle levels and the magic gaps depend on the relative neutron-excess of the nuclei.

I.) The more aestetic question is, how the single-particle levels of neutrons and protons depend on the number of nucleons A and the central neutron excess $\bar{\delta} := (\rho_n - \rho_p)/(\rho_n + \rho_p)$ (or the global asymmetry I : = (N - Z)/A respectively). First, one has to sort out the more or less "trivial" effects:

α) the single-particle levels reflect the depth of the single-particle(Woods-Saxon) potential $V_0(A,I)$ being proportional to V_0, if they are more bound than some 4 MeV. The function V_0 (A,I) e.g. for neutrons is given in [1].

β) the single-particle levels reflect the absolute size of the nucleus, being more bound for heavier and thus larger nuclei. For low-lying levels, where the Woods-Saxon might be approximated by a box with infinitely steep walls at the equivalent sharp radius R_V $(\frac{N}{Z})$, we get the estimate for the average mean of the kinetic energy of e.g. a neutron-level of eigenstate of number n,

$$(1) \quad t_n \simeq \frac{\hbar^2}{2m} \cdot \left(3\pi^2 \cdot n \cdot V^{-1}\right)^{2/3} \cdot \left\{ 1 + \frac{\pi}{4} \frac{S}{V} \cdot \left(3\pi^2 n V^{-1}\right)^{-1/3} + \cdots \right\}$$

in second order in $A^{-1/3}$, see [2], where V and S are the volume, and the surface of the neutrons of the nucleus, respectively. Taking the relations for the potential radius R_V (N) of [1] and with $R_\rho = r_0 A^{1/3}$ we get

$$(2) \quad \begin{aligned} V(N) &= \frac{4\pi}{3} R_V^3(N) = \frac{4\pi}{3} r_0^3 \cdot A \cdot \left\{ 1 + \frac{1.64 + \frac{8}{3}\bar{\delta}^2}{r_0 A^{1/3}} + \cdots \right\}, \\ S/V &= \frac{3}{r_0 A^{1/3}} \cdot \left\{ 1 - \frac{0.82 + \frac{4}{3}\bar{\delta}^2}{r_0 A^{1/3}} + \cdots \right\}, \end{aligned}$$

n-skin thickness is proportional to $\bar{\delta}$. This effect, of course, can be exhibited only, if the neutron-levels for isotones are plotted (see fig. 2 , ref. [1]), where the same levels define the Fermi-surface, whereas for isotopes the n-levels just cross the Fermisurface at one nucleus.

For protons one would not expect at all such an ℓ-dependence, since the nuclei of interest have a neutron-skin according to the Droplet Model, thus the proton-density distribution being always inside the neutrons.

γ) The question, what the neutron-separation energies are for n-rich nuclei, and where the n-dripline is located in the N — Z plane. The predictions of ref. [1] , that ^{28}O and ^{60}Ca are more bound, but ^{60}Ca is less bound compared to e.g. the mass formula of Myers and Swiatecki, the shell term of which does not take the ℓ-dependence of the magic gaps into account, is established by the laborous calculations of the RBHF method of [4] with a density dependent Hamiltonian, see table, ref. [5].

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