

Preparatory course of Mathematics

1 Target group

Beginner students in all courses of physics.

2 Objective

Revision of the mathematical knowledge from school which is necessary at the beginning of a study in physics. Depending on the school, gaps stemming from different curricula are filled.

3 Organization

The duration of the preparatory course is two weeks. There are two lectures of 90 minutes every day – one in the morning, one in the afternoon. After the lectures, exercises are worked out and discussed in smaller groups. The course is held in german.

4 Contents (orientation)

1. Basics

- Numerical systems, basic arithmetic methods, fractional arithmetics with variables
- Power laws, logarithmic laws, term conversions

2. Equations

- Elementary trigonometry, $\sin(x)$, $\cos(x)$, laws of sines and cosines
- Equations, linear equations
- Quadratic equations
- Equations with roots and absolute values
- Inequalities
- Systems of linear equations, matrices, determinants, Law of Sarrus and Cramer's Law for solving systems of linear equations

3. Functions

- Elementary functions: polynomials, $\sin(x)$, $\cos(x)$, e^x , $\ln(x)$, $\sinh(x)$, $\cosh(x)$, $\operatorname{arcsinh}(x)$, ...: important properties, connections
- Functions with parameters, determination from known function values
- Functions in several variables

4. Vector analysis

- Vectors
- Coordinate systems
- Scalar product, vector product
- Linear maps

5. Complex numbers

- Calculating with complex numbers

- Representations (cartesian, polar)
- Functions with complex arguments (e.g. $\sin(x + iy)$)
- Connections between functions with complex arguments

6. Differential calculus

- Differential quotient, physical interpretation
- Laws of derivation (sum rule, product rule, quotient rule, chain rule)
- Derivations of vectors
- Taylor expansion

7. Integral calculus

- Definition of an integral, elementary properties
- Integration of elementary functions
- Laws for integration (substitution, partial integration)

5 Examples of exercises

1. Determine the derivative of

$$f_1(x) = \cos(x^3), \quad f_2(x) = \cos^3(x), \quad f_3(x) = x^x.$$

2. Determine an antiderivative of $f(x) = \sin(x) \exp(-ax)$.

3. Determine the limit

$$\lim_{x \rightarrow 0} \frac{\arcsin(x)}{x}.$$

4. Write $\left(\frac{1-i}{1+i}\right)^{10}$ in the form $a + ib$.

5. Calculate

$$\det \begin{pmatrix} 1 & 0 & 3 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{pmatrix}.$$

6. Determine a matrix M which describes a rotation of angle $\pi/4$ around the z -axis

7. Calculate the series expansion of $g(x) = \exp(-ax^2) \sin(x)$ up to third order.